Real World Activities Involving Volume, Surface Area, and Trigonometry

Objectives

Students will solve real world situations involving volumes and trigonometry.

Core Learning Goals

2.2.2 The student will solve problems using two-dimensional figures and/or right-triangle trigonometry

2.3.2 Students will use techniques of measurement and will estimate, calculate and/or compare perimeter, circumference, area, volume, and surface area of two- and three-dimensional figures and their parts.

Materials Needed

Worksheets, graph paper, popsicle sticks, paper, tape, calculator, HSA formula reference sheet and poster board

Pre-requisite Concepts Needed

Students will need to be able to determine the volume of a rectangular prism, cylinder and cone. Students will also need to be familiar with sine and cosine relationships in a right triangle.

Approximate Time

One 45-minute lesson

Lesson Plan

Warm Up/Opening Activity

Worksheet: **Warm-up**

Answers: 1. 86.6
2. 16/3 Be sure students leave answer as an improper fraction

Development of Ideas

The students will solve problems in cooperative groups. Each group will be assigned 6 problems; the last problem is to find patterns in a trigonometry table. The theme of the assignment is to earn a camping badge. The teacher can assign points for each question a group gets right or a prize for the group that gets all of the problems correct. Each group could display or present one of their solutions.

Teachers need to be aware that the answers can vary, depending on how the students round their answers. The problems can be done in class or started in class and finished for homework.

Answers

Station 1

(Volume of the ice chest - volume of the ice) ÷ the volume of the cone
= the number of cones to be purchased

Volume of the ice chest: $V = lwh$
$V = 12 \cdot 22 \cdot 11 = 2904 \text{ in}^3$

Volume of the ice: $V = Bh$
$V = \pi \cdot 5^2 \cdot 18$
$= 1413.72 \text{ in}^3$

Volume of the cone: $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 1.5^2 \cdot 5 = 11.78 \text{ cm}^3$

$2904 - 1413.72 = 1490.28$
$1490.28 \div 11.78 \approx 126$ cones

Answers can vary depending on rounding

Development of Ideas (Continued)

Answers (Continued)

Station 2

SA of the cone shaped tent (no canvas on ground)

$= \pi r l = \pi \cdot 4 \cdot 8.94 = 112.3$ ft² $\qquad l = \sqrt{8^2 + 4^2} \approx 8.94$

SA of the sq. shaped pyramid

$= \frac{1}{2} l p = \frac{1}{2} \cdot 8.94 \cdot 32 = 143.04$ ft²

SA Leader's Tent

SA of the tent sides and bottom

$= Ph + s^2 = 32 \cdot 5 + 8 \cdot 8 = 160 + 64 = 224$ ft²

SA of the top – the top of the tent is a triangular prism. You can calculate the area of each face, remember there is no bottom to the top of the tent.

SA = $2(.5)(3)(8)$*triangles* + $2(5)(8)$*rectangles* = 104 ft²

The total canvas needed for the leader's tent is 328 ft²

The leader's tent has the most room inside.

Volume of the cone shaped tent = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi 4^2 \cdot 8 = 134.04$ ft³

Volume of the square shaped pyramid = $\frac{1}{3}Bh = \frac{1}{3}(64)(8) = 170.7$ ft³

Volume of the leader's tent = lwh + Bh (top)

$= 8 \cdot 8 \cdot 5 + .5(8)(3)(8) = 320 + 96 = 416$ ft³

Station 3

C = $2\pi r \div 2$ (semicircle frame) = $\pi r = \pi \cdot 114 = 358.14$ in

(change ft. to inches)

358.14 divided by (5.25 + .25 gap) = 65.11 boards

You will need 66 boards times \$3.47 = \$229.02 (cost for the boards)

The frame is a semicircle, so the formula is $\pi r = (9.5)(\pi) \approx$ 29.8 feet

Charge is 30 feet times .37 = \$11.10

\$11.10 · 2 frames = cost for frames is \$22.20

Total cost for the bridge is \$22.20 + \$229.02 = \$251.22

Development of Ideas (Continued)

Answers (Continued)

Station 4

There are many answers to this question. One way to get the answer is to start with a certain length ladder. Suppose, you have a 25-foot ladder, then you would build the look out stand 19.15 feet off the ground. Sin 50 = height of the lookout stand divided by the length of the ladder.

Length of the ladder	Height of the lookout stand
20	15.32
25	19.15
30	22.98

Station 5

To solve this problem, find the volume of the cylinder divided by the volume of the cup = to the number of the cups of water you can get out of the cooler.

$V = Bh = \pi r^2 h = \pi \cdot 9^2 \cdot 36 \approx 9160.88$

$V = \frac{1}{3}Bh = \frac{1}{3}\pi (1.25^2)(3) \approx 4.91$

$9160.88 \div 4.91 \approx 1866$ cups of water

Problem 6

Angle	Sine	Value	Height of the tree	Length of line of sight
0°	Sin 0°	0	25	--
10°	Sin 10°	.174	25	143.97 ft
20°	Sin 20°	.342	25	73.1 ft
30°	Sin 30°	.5	25	50 ft
40°	Sin 40°	.643	25	38.89 ft
50°	Sin 50°	.766	25	32.64 ft
60°	Sin 60°	.866	25	28.87 ft
70°	Sin 70°	.940	25	26.60 ft
80°	Sin 80°	.985	25	23.39 ft
90°	Sin 90°	1	25	25 ft

Development of Ideas (Continued)

Answers (Continued)

Problem 6 (Continued)

Angle	Cos.	Value	Height of the tree	Length of line of sight
0°	Cos 0°	1	25	25 ft
10°	Cos 10°	.985	25	25.39 ft
20°	Cos 20°	.940	25	26.6 ft
30°	Cos 30°	.866	25	28.87 ft
40°	Cos 40°	.766	25	32.64 ft
50°	Cos 50°	.643	25	38.89 ft
60°	Cos 60°	.5	25	50 ft
70°	Cos 70°	.342	25	73.10 ft
80°	Cos 80°	.174	25	143.97 ft
90°	Cos 90°	0	25	--

Some observations about the charts:

(1) As the angle measure increases the values of the cosine decreases. With the sine values, the opposite occurs. As the angle measure increases the sine values increase.

(2) When the angle measures are complementary, the values of the sine and the cosine are equal.

(3) As the angle measures increase, the length of the line of sight decreases when the sine is used. The opposite occurs when the cosine is used.

Closure

1. How is the formula for volume of a pyramid different from the formula for the volume of a cylinder?

   Answer: The volume of a pyramid is $\frac{1}{3}$ of the area of the base times the height while the volume of a cylinder is the area of the base times the height.

Closure (Continued)

2. What is the difference between the height of a cone and the slant height of a cone?

   Answer: The height of a cone is the perpendicular distance from the vertex of the cone to the base. The slant height is the distance from the vertex along the lateral surface to the edge of the base.

3. In a triangular prism, what is the shape of the sides?

   Answer: In a triangular prism the sides are rectangles.

Homework

Worksheet: **Homework**

Answers:

1. The ice chest is given in inches, so the students need to covert to centimeters (2.54 cm = 1 inch).

   The volume of the ice chest is $(12)(22)(11)(2.54^3) = 47588 \text{ cm}^3$

   Subtract the volume of the ice $(18)(18)(20) = 6480 \text{ cm}^3$

   Divide by the volume of the Egyptian Ice $\frac{1}{3}(36)(7) = 84$

   The answer is 489 containers of Egyptian Ice.

2. First you must change feet to inches, so 19 feet is 228 inches, then divide the distance across the stream, 228, by the width and gap of the board, 5.5 inches. You have to buy 42 boards at $3.47 a board. The cost for the boards will be $145.74. The 2 frames which cost 37 cents a linear foot, so the cost of the frames will be $2 \cdot 19 \cdot 0.37 = \$14.06$. The grand total of the project is $159.80.

3. In this problem you can have many different answers depending on how high up the tree you want to put the lookout stand and the length of the ladder. A few answers are:

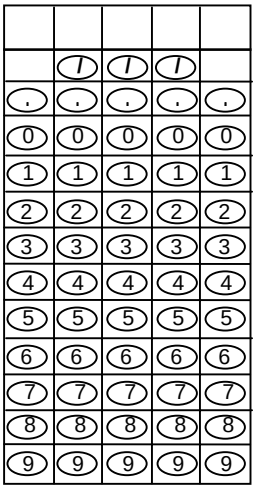
Length of the ladder	Height of the lookout stand
20 feet	18.79 feet
25	23.49
31.92	30

   Sin 70° = height of the lookout stand divided by the length of the ladder.

Warm-up

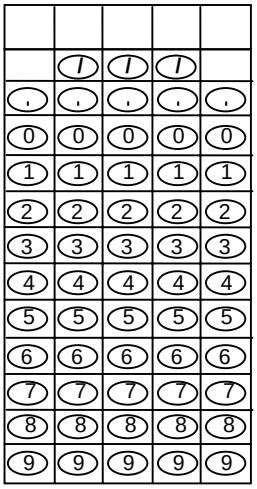
1. The angle of elevation from the ground to the top of the building is 60 degrees. If the vertex of the angle is 50 feet from the base of the building, how tall, in feet, is the building?

   Grid your answer, to the nearest tenth of a foot, in the grid at the right.



2. A pyramid has a square base with sides of 2 centimeters and a height of 4 centimeters. What is the volume, in cubic centimeters, of the pyramid?

   Grid your answer as a fraction, in the grid at the right.



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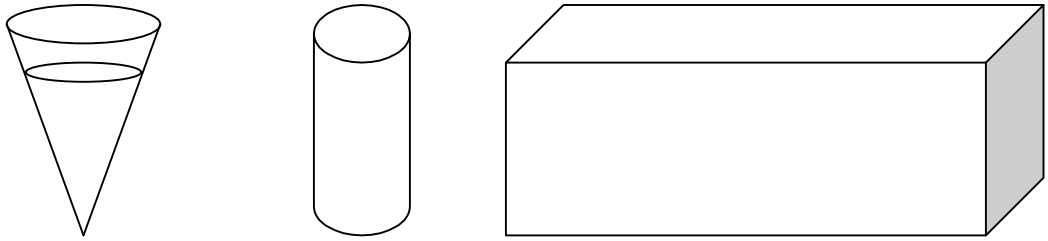
Activity Sheet

Troop 407 is going on a camping trip and their leader has given the scouts of troop 407 a challenge. They have to earn 6 merit points to receive their camping badge. The leader will divide the troop into groups of 3 or 4. To get in the spirit of group work, each group should choose a name. Each group will try to solve each of the 6 problems. To earn the camping badge the group must have the correct answer, with the required explanation or justification, to each of the 6 stations, earning a merit point for each station.

Station 1

Your group has been assigned the task of bringing snacks for the trip. You decided to bring ice cream cones, the cones that have the ice cream on top wrapped in paper and comes to a point, a perfectly shaped cone. Each cone with ice cream is 5 inches tall and has a diameter of 3 inches. You must figure out how many cones you should buy, if the rectangular prism cooler you are going to store them in is 12 inches x 22 inches x 11 inches. You will also put a bag of ice in the cooler. The bag of ice is shaped like a cylinder that is 10 inches in diameter and 18 inches tall. Use mathematics to explain how you determine out how many cones you will need. Use words, symbols or both in your explanation.



Ice cone bag of ice cooler

Note: The figures are not drawn to scale.

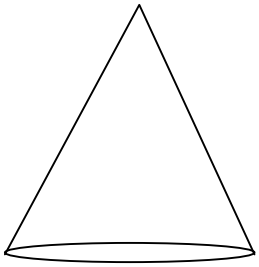
Station 2

To get a merit point, your group must make a tent in the shape of a cone or in the shape of a square pyramid. Your tent will not have canvas on the bottom. The coned shaped tent has a diameter of 8 feet and height of 8 feet. The square pyramid shaped tent has a base with sides of 8 feet and a height of 8 feet. The leaders will use a tent shaped like a square prism with each face a rectangle with sides of 8 feet and height of 5 feet. The top of the tent is shaped like a triangle prism with a height of 3 feet. The leader's tent has canvas on the bottom. How much canvas is needed for each tent (see drawings on next page)?

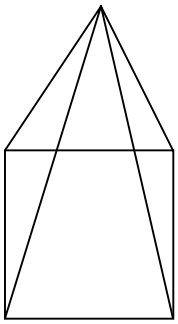
Activity Sheet (Continued)

Station 2 Continued

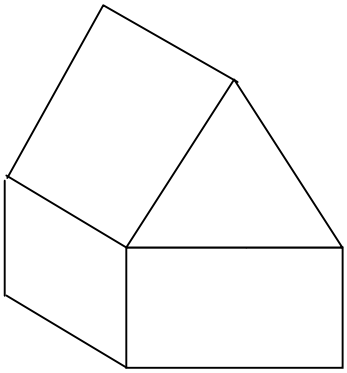
Use mathematics to explain how you determine your answer. Use words, symbols, or both in your explanation. Which of the three tents has the most room inside? Use mathematics to explain how you determine your answer. Use words, symbols, or both in your explanation.



Cone shaped tent



Square pyramid tent

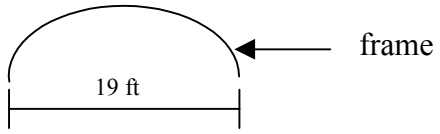


Tent – triangular prism top
rectangular prism bottom

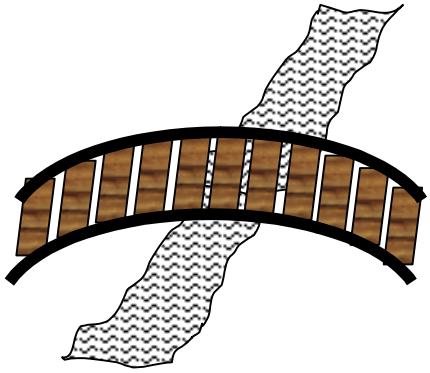
Note: The figures are not drawn to scale.

Station 3

The scouts want to build a bridge across a stream. Your group has to calculate the number of boards needed for the project and the total cost. Unfortunately, there is a tree in the stream so you have to build a bridge in shape of a semicircle. (See picture below) The semicircular bridge will have a diameter of 19 feet. Each board is 5.25 inches wide with a .25 inch gap between the boards. The boards cost $3.47 each. The frames, for each side of the bridge, come in the shape of a semicircle. The frames cost $.37 a linear foot.



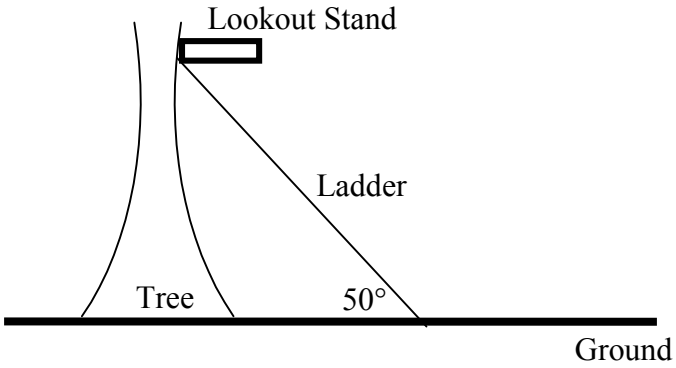
What is the total number of boards needed to build the bridge and what is the total cost of the bridge? Use mathematics to explain how to determine your answers. Use words, symbols or both in your explanation.



Activity Sheet (Continued)

Station 4

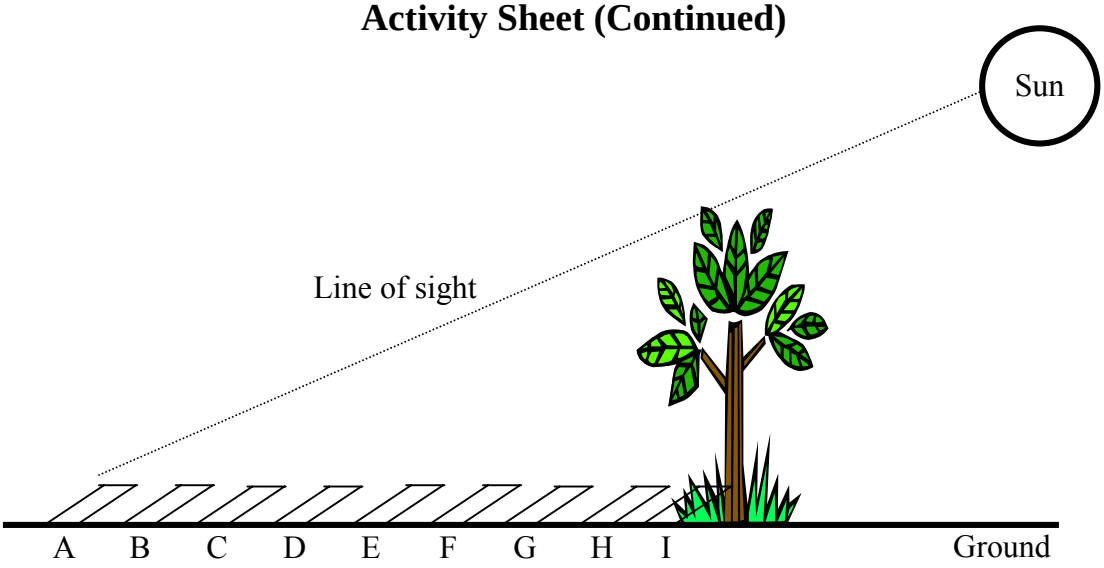
The next merit point can be earned by building a lookout stand in a tree. You have to determine how far up the tree the stand should go. To climb up to the lookout stand you are going to attach a ladder to the bottom of the lookout stand where the tree meets the look out stand (see figure below). The ladder must make a 50-degree angle with the ground so it will meet all safety requirements. You must determine how long the ladder must be and how far up the tree you can build the lookout stand with an angle of 50 degrees. You do not have a protractor to measure the angle, but you do have a calculator that has all your trigonometry functions and a tape measure. Propose three possible solutions to this situation. Use mathematics to explain how you determine your answers. Use words, symbols or both in your explanation.



Station 5

On one of the days at camp, your troop is going on a 10-mile hike. To earn another merit point you must determine how many cups of water the water cooler holds. The water cooler is shaped like a cylinder, 18 inches in diameter and 3 feet high. The cups are shaped like a cone 3 inches high and 2.5 inches in diameter. How many cups of water will the cooler hold? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.

Activity Sheet (Continued)



Station 6

Your final task is to find patterns in a trigonometric table. You and eight of your friends decide to sleep under the stars on the west side of a tree that is 25 feet tall. Right triangles are formed by the tree, the ground, and the distance to the top of the tree from each camper. As the sun rises in the east, it will shine in each campers eyes starting with camper A and every half-hour move to the next camper. The angle of elevation from the camper to the top of the tree increases by 10 degrees each time it moves to the next camper. Complete the chart below to calculate the length of the line of sight from each camper to the top of the tree.

Camper	Angle	Sine	Value	Height of the tree	Length of line of sight
	0°	Sin 0°		25	
A	10°	Sin 10°		25	
B	20°	Sin 20°		25	
C	30°	Sin 30°		25	
D	40°	Sin 40°		25	
E	50°	Sin 50°		25	
F	60°	Sin 60°		25	
G	70°	Sin 70°		25	
H	80°	Sin 80°		25	
I	90°	Sin 90°		25	

What pattern(s) do you see in the chart?

Activity Sheet (Continued)

In this problem you are still looking for the distance from the top of the tree to the eyes of the campers on the ground. This time you are going to use the angle formed by the tree and the line of sight to the eyes of the camper. The height of the tree remains at 25 feet. Complete the table to calculate the line of sight from each camper to the top of the tree.

Campers	Angle	Cos.	Value	Height of the tree	Length of line of sight
	0°	Cos 0°		25	
A	10°	Cos 10°		25	
B	20°	Cos 20°		25	
C	30°	Cos 30°		25	
D	40°	Cos 40°		25	
E	50°	Cos 50°		25	
F	60°	Cos 60°		25	
G	70°	Cos 70°		25	
H	80°	Cos 80°		25	
I	90°	Cos 90°		25	

What pattern(s) do you see in the chart?

If you compare both charts what pattern(s) do you see?

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Homework

1. Suppose you brought Egyptian Ice instead of the ice cream cones in Station 1. The Egyptian Ice, you guessed it, is in the shape of a pyramid. The dimensions of container are 6 cm by 6 cm base, with a height of 7 cm. The chest is 12 inches x 22 inches x 11 inches and the ice package is shaped like a rectangular prism 18 cm x 18 cm x 20 cm. Use 1 inch = 2.54 cm. How many Egyptian Ice containers can you fit in the cooler? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.

2. Suppose the bridge in Station 3 was built straight across the stream instead of a semicircle. How much money will it cost you to build this bridge? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.

3. Suppose the safety requirements for the ladder in Station 4 was changed from 50° to 70°. Calculate the possible lengths of the ladder and the distance from the base of the tree. Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.